FULL PORTION TEST

DATE: **TIME: 3 hrs**

CLASS: XII **Max.Marks:200**

SECTION – A **[40x1=40]**

N.B. i) Answer all the questions.
ii) Each questions carries one mark.

1. If A is a scalar matrix with scalar k≠0, of order 3, then A^{-1} is

1) $\frac{1}{k^2}I$ 2) $\frac{1}{k^3}I$ 3) $\frac{1}{k}I$ 4) kI

2. If the matrix $\begin{bmatrix} -1 & 3 & 2 \\ 1 & k & -3 \\ 1 & 4 & 5 \end{bmatrix}$ has an inverse then the values of k

1) k is any real number 2) k = -4 3) k≠ -4 4) k≠4

3. If $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$,then (adj A)A =

1) $\begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{bmatrix}$ 2) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 3) $\begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix}$ 4) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

4. In the system of 3 linear equations with three unknowns, if $\Delta = 0, \Delta_x = 0, \Delta_y = 0, \Delta_z = 0$ and atleast one 2x2 minor of $\Delta \neq 0$ then the system is
1) consistent 2) inconsistent
3) consistent and the system reduces to two equations
4) consistent and the system reduces to a single equation.

5. If $\vec{a}$ is a non-zero vector and m is a non-zero scalar then m$\vec{a}$ is a unit vector if

1) m = # 1 2)a = $|m|$ 3) a = $\frac{1}{|m|}$ 4) a =1

6. If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them, then $(\vec{a}+\vec{b})$ is a unit vector if 1) $\theta = \frac{\pi}{3}$ 2) $\theta = \frac{\pi}{4}$ 3) $\theta = \frac{\pi}{2}$ 4) $\theta = \frac{2\pi}{3}$

7. The area of the parallelogram having a diagonal $3\vec{i}+\vec{j}-\vec{k}$ and a side $\vec{i}-3\vec{j}+4\vec{k}$ is

1) $10\sqrt{3}$ 2) $6\sqrt{30}$ 3) $\frac{3}{2}\sqrt{30}$ 4) $3\sqrt{30}$

8. If $\vec{p}, \vec{q}$ and $\vec{p}+\vec{q}$ are vectors of magnitude λ then the magnitude of $|\vec{p}-\vec{q}|$ is

1) 2λ 2) $\sqrt{3}\lambda$ 3) $\sqrt{2}\lambda$ 4) 1

9. If $\vec{a}=\vec{i}-2\vec{j}+3\vec{k}$ and $\vec{b}=3\vec{i}+\vec{j}+2\vec{k}$ then a unit vector perpendicular to $\vec{a}$ and $\vec{b}$ is

1) $\frac{\vec{i}+\vec{j}+\vec{k}}{\sqrt{3}}$ 2) $\frac{\vec{i}-\vec{j}+\vec{k}}{\sqrt{3}}$ 3) $\frac{-\vec{i}+\vec{j}+2\vec{k}}{\sqrt{3}}$ 4) $\frac{\vec{i}-\vec{j}-\vec{k}}{\sqrt{3}}$

10. The vector equation of a plane whose distance from the origin is p and perpendicular to a unit vector $\hat{n}$ is 1. $\vec{r}\,.\,\vec{n}=p$ 2. $\vec{r}\,.\,\hat{n}=q$ 3. $\vec{r}\times\vec{n}=p$ 4. $\vec{r}\,.\,\hat{n}=p$

11. The points z_1, z_2, z_3, z_4 in the complex plane are the vertices of a parallelogram taken in order if and only if

1) $z_1+z_4=z_2+z_3$ 2) $z_1+z_3=z_2+z_4$ 3) $z_1+z_2=z_3+z_4$ 4) $z_1-z_2=z_3-z_4$

12. If $z_n=\cos\frac{n\pi}{3}+i\sin\frac{n\pi}{3}$ then $z_1z_2z_3...z_6$ is

1) 1 2) -1 3) i 4) –i

13. If a = cos α - i sin α, b = cos β - i sinβ, c = cos γ - i sinγ □ □ then($a^2 c^2 - b^2$) / abc is

1) $\cos 2(\alpha-\beta+\gamma)+i\sin 2(\alpha-\beta+\gamma)$ 2) $-2\cos(\alpha-\beta+\gamma)$

3) $-2i\sin(\alpha-\beta+\gamma)$ 4) $2\cos(\alpha-\beta+\gamma)$

14. Identify the correct statement

1. Sum of the moduli of two complex numbers is equal to their modulus of the sum
2. Modulus of the product of the complex numbers is equal to the sum of their moduli
3. Arguments of the product of two complex numbers is the product of their arguments.
4. Arguments of the product of two complex numbers is equal to sum of their arguments.

15. The vertex of the parabola $x^2 = 8y - 1$ 1) $\left(-\frac{1}{8},0\right)$ 2) $\left(\frac{1}{8},0\right)$ 3) $\left(0,\frac{1}{8}\right)$ 4) $\left(0,-\frac{1}{8}\right)$

16. The length of the semi-major and the length of semi-minor axis of the ellipse $\frac{x^2}{144}+\frac{y^2}{169}=1$ is

1) 26, 12 2) 13, 24 3) 12, 26 4) 13, 12

17. The sum of the distance of any point on the ellipse $4x^2+9y^2 = 36$ from $(\sqrt{5},0)$ and $(-\sqrt{5},0)$ is

1) 4 2)8 3)6 4)18

18. The condition that the line $lx+my+n=0$ may be a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

1. $al^3+2alm^2+m^2n=0$
2. $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{(a^2+b^2)^2}{n^2}$
3. $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{(a^2-b^2)^2}{n^2}$
4. $\frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{(a^2+b^2)^2}{n^2}$

19. The angle between the curves $\frac{x^2}{25}+\frac{y^2}{9}=1$ and $\frac{x^2}{8}-\frac{y^2}{8}=1$ is

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{2}$

20. The parametric equation of the curve $x^{2/3}+y^{2/3}=a^{2/3}$ are

1) $x = a\sin^3$ □ , $y = a\cos^3$ □ 2) $x = a\cos^3$ □ , $y = a\sin^3$ □

3) $x = a^3\sin$ □ , $y = a^3\cos$ □ 4) $x = a^3\cos$ □ , $y = a^3\sin$ □

21. The value of 'a' so that the curves $y = 3e^x$ and $y=\frac{a}{3}e^{-x}$ intersect orthogonally is

1) -1 2) 1 3) $\frac{1}{3}$ 4) 3

22. A continuous graph $y = f(x)$ is such that $f'(x) \to \infty$ as $x \to x_1$ at (x_1, y_1). Then $y = f(x)$ has a

1. vertical tangent y = x_1
2. horizontal tangent x = x_1
3. vertical tangent x = x_1
4. horizontal tangent y = y_1

23. An asymptote to the curve $y^2 (a + 2x) = x^2 (3a - x)$ is

1) x = 3a 2) x = $-\frac{a}{2}$ 3) x = $\frac{a}{2}$ 4) x = 0

24. The curve $y^2 (2 + x) = x^2 (6 - x)$ exists for

1. $-2 < x \le 6$ 2. $-2 \le x \le 6$ 3. $-2 < x < 6$ 4. $-2 \le x < 6$

25. The area of the region bounded by the graph of y = sinx and y = cosx between x = 0 and x = $\frac{\pi}{4}$ is 1) $\sqrt{2}+1$ 2) $\sqrt{2}-1$ 3) $2\sqrt{2}-2$ 4) $2\sqrt{2}+2$

26. The volume generated when the region bounded by y = x, y =1, x = 0 is rotated about y-axis is

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{2\pi}{3}$

27. The curved surface area of a sphere of radius 5, intercepted between two parallel planes of distance 2 and 4 from the centre is

1) 20π 2) 40π 3) 10π 4) 30π

28. $\int_0^a f(x)dx + \int_0^a f(2a-x)dx =$

1. $\int_0^a f(x)dx$ 2. $2\int_0^a f(x)dx$ 3. $\int_0^{2a} f(x)dx$ 4. $\int_0^{2a} f(a-x)dx$

29. The degree of the differential equation $\sqrt{1+\left(\frac{dy}{dx}\right)^{1/3}} = \frac{d^2y}{dx^2}$ is

1) 1 2) 2 3) 3 4) 6

30. The amount present in a radioactive element disintegrates at a rate proportional to its amount. The differential equation corresponding to the above statement is (k is negative)

1) $\frac{dp}{dt} = \frac{k}{p}$ 2) $\frac{dp}{dt} = kt$ 3) $\frac{dp}{dt} = kp$ 4) $\frac{dp}{dt} = -kt$

31. If $f'(x) = \sqrt{x}$ and $f(1) = 2$, then $f(x)$ is

1) $-\frac{2}{3}(x\sqrt{x}+2)$ 2) $\frac{3}{2}(x\sqrt{x}+2)$ 3) $\frac{2}{3}(x\sqrt{x}+2)$ 4) $\frac{2}{3}x(\sqrt{x}+2)$

32. Identify the incorrect statement.

1. The order of a differential equation is the order of the highest order derivative occurring in it.
2. The degree of the differential equation is the degree of the highest order derivative which occurs in it. (The derivatives are free from radicals and fractions).
3. $\frac{dy}{dx} = \frac{f_1(x,y)}{f_2(x,y)}$ is the first order and first degree homogeneous differential equation.
4. $\frac{dy}{dx} + xy = e^x$ is a linear differential equation in x.

33. In the set of real numbers R, an operation * is defined by a * b = $\sqrt{a^2+b^2}$ Then the value of (3 * 4)* 5 is

1)5 2)$5\sqrt{2}$ 3)25 4)50

34. In the multiplicative group of n^{th} roots of unity, the inverse of ω^k is (k < n)

1) $\omega^{1/k}$ 2) ω^{-1} 3) ω^{n-k} 4) $\omega^{n/k}$

35. In the set of integers under the operation * defined by a * b = a + b -1, the identity element is
   1)0 2)1 3)a 4)b

36. In congruence modulo5, $\{x \in z / x = 5k+2,\ k \in z\}$ represents
   1. $[0]$ 2. $[5]$ 3. $[7]$ 4. $[2]$.

37. Variance of the random variable X is 4. Its mean is 2. Then E(X^2) is
   1)2 2)4 3)6 4)8

38. In 5 throws of a die, getting 1 or 2 is a success. The mean number of successes is
   1. $\frac{5}{3}$ 2. $\frac{3}{5}$ 3. $\frac{5}{9}$ 4. $\frac{9}{5}$

39. If the mean and standard deviation of a binomial distribution are 12 and 2. Then the value of its parameter p is
   1. $\frac{1}{2}$ 2. $\frac{1}{3}$ 3. $\frac{2}{3}$ 4. $\frac{1}{4}$

40. A discrete random variable X has probability mass function p(x), then
   1. $0 \le p(x) \le 1$ 2. $p(x) \ge 0$ 3. $p(x) \le 1$ 4. $0 < p(x) < 1$

SECTION - B

N.B: i) Answer any TEN questions. **[10x6=60]**

ii) Question 55 is compulsory and choose any NINE questions from the remaining

(iii) Each questions carries 6 marks.

41. Find the rank of the matrix $\begin{bmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{bmatrix}$.

42. Solve the following non-homogeneous system of linear equations by determinant method

4x+5y = 9, 8x+10y = 18.

43. Show that $\left[\vec{a}+\vec{b}+\vec{c} \quad \vec{b}+\vec{c} \quad \vec{c}\right] = \left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

44. (i) Find the direction cosines of the line joining (2, -3, 1) and (3, 1, -2).
   (ii) Find the angle between the following planes 2x-3y+4z = 1 and -x+y = 4.

45.(i) Express the following complex numbers in polar form -1-i .

(ii) Find all the values of the following: $i^{\frac{1}{3}}$

46.Prove that $(1+i)^{4n}$ *and* $(1+i)^{4n+2}$ are real and purely imaginary respectively.

47. Determine for which values of x, the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is increasing and for which it is decreasing. Also determine the points where the tangents to the graph of the function are parallel to the x-axis.

48. If $V = ze^{ax+by}$ and z is a homogeneous function of degree n in x and y prove that

$$x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} = (ax+by+n)V\ .$$

49 (i) Evaluate : $\int_0 \sqrt{a^2 - x^2}\,dx$

(ii) Evaluate $\int_{-1}^{1} \sin x \cos^4 x\,dx$

50. Form the differential equation by eliminating arbitrary constants given in brackets for

$y = Ae^{2x}\cos(3x + B)$, {A, B}

51. Show that $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

52. State and prove reversal law on inverses of a group.

53. For the p.d.f $f(x) = \begin{cases} cx(1-x)^3, 0 < x < 1 \\ 0, \quad elsewhere \end{cases}$ Find (i) the constant c (ii) $P\left(x < \frac{1}{2}\right)$

54. The probability of success of an event is p and that of failure is q. Find the expected number of trials to get a first success.

55. (i) Find the equations of the two tangents that can be drawn from the point (5, 2) to the ellipse $2x^2+7y^2 = 14$.

(OR)

(ii) A cylindrical hole 4 mm in diameter and 12mm deep in a metal block is rebored to increase the diameter to 4.12 mm. Estimate the amount of metal removed.

Section – C

N.B. (i) Answer any TEN questions.

(ii) Question No: 70 is compulsory and choose any NINE questions from remaining.

(iii) Each questions carries 10 marks. [10x10=100]

56. Investigate for what values of λ, μ the simultaneous equations

x+y+z = 6, x+2y+3z =10, x+2y+ λ z = μ

57. verify $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a}\ \vec{b}\ \vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$, if $\vec{a} = \vec{i}+\vec{j}+\vec{k}$, $\vec{b} = 2\vec{i}+\vec{k}$, $\vec{c} = 2\vec{i}+\vec{j}+\vec{k}$, $\vec{d} = \vec{i}+\vec{j}+2\vec{k}$

58. Show that the lines $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x-2}{1} = \frac{y-1}{2} = \frac{-z-1}{1}$ intersect and find their point of intersection.

59. P represents the variable complex number z . Find the locus of P, if $\text{Im}\left[\frac{2z+1}{iz+1}\right] = -2$

60. Find the axis, vertex, focus, equation of directrix, latus rectum , length of the latus rectum for the parabola $y^2+8x-6y+1=0$ and hence sketch the graph.

61. A cable of a suspension bridge hangs in the form of a parabola when the load is uniformly distributed horizontally. The distance between two towers is 1500 ft, the points of support of the cable on the towers are 200ft above the road way and the lowest point on the cable is 70ft above the roadway. Find the vertical distance to the cable (parallel to the roadway) from a pole whose height is 122ft.

62. Show that the line 5x+12y = 9 touches the hyperbola $x^2-9y^2=9$ and its point of contact.

63. Two sides of a triangle have length 12 m and 15m. The angle between them is increasing at a rate of 2°/min. How fast is the length of third side increasing when the angle between the sides of fixed length is 60°?

64. Find the local maximum and minimum values of 't+cost'.

65. Using Euler's theorem, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u,\ if\ \ u=\sin^{-1}\left(\frac{x-y}{\sqrt{x}+\sqrt{y}}\right)$.

66. Show that the surface area of the solid obtained by revolving the arc of the curve

y = sinx from x = 0 to x = π about x-axis is $2\pi\left[\sqrt{2}+\log(1+\sqrt{2})\right]$.

67. Solve the differential equation dy= x3 dy+3x2ydx+secx(secx+tanx)dx.

68. Show that the set of four functions f_1, f_2, f_3, f_4 on the set of non-zero complex numbers C- { 0 } defined by $f_1(z)=z,\ f_2(z)=-z,\ f_3(z)=\frac{1}{z},\ f_4(z)=-\frac{1}{z},\ \forall z\in C-\{0\}$ forms an abelian group with respect to the composition of functions.

69. The probability density function of a random variable *x* is

$$f(x)=\begin{cases} kx^{\alpha-1}e^{-\beta x^{\alpha}}, & x,\alpha,\beta>0 \\ 0, & elsewhere \end{cases}$$

Find (i) k (ii) P(X>10)

70. a) Find the area of the curve $y^2=(x-5)^2(x-6)$

(i) the area between x = 5 and x = 6 (ii) between x = 6 and x = 7

(OR)

Solve $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$ given that y=1, where x=0.