Register Number | | | | |

MATHEMATICS

Time Allowed : **3** Hrs — Maximum Marks : **200**

Instrutions: (1) Check the question paper for fairness of printing. If there is any lack of fairness inform the Hall Supervisor immediately.

(2) Use Blue or Black ink to write and pencil to draw diagrams

PART – A

Note: (i) **All** Questions are **compulsory.**

(ii) Choose the most suitable answer from the given four alternatives and write the **option code** and the **corresponding answer**

40×1= 40

1 The curve $y^2(x-2)=x^2(1+x)$ has

a) an asymptote parallel to x-axis b) an asymptote parallel to y-axis

c) asymptotes parallel to both axes d) no asymptotes

2 In which region the curve $y^2 (a+x) = x^2 (3a-x)$ does not lie ?

a) $x > 0$ b) $0 < x < 3a$. c) $x < -a$ and $x > 3a$ d) $-a < x < 3a$

3 The area of the region bounded by the graph of y = sin x and y = cos x between x = 0 and x = $\pi/4$ is.

a) $\sqrt{2}+1$ b) $\sqrt{2}-1$ c) $2\sqrt{2}+1$ d) $2\sqrt{2}+2$

4 Volume of solid obtained by revolving the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about major and minor axes are in the ratio.

a) $b^2 : a^2$ b) $a^2 : b^2$ c) a : b d) b : a

5 The curved surface area of a sphere of radius 5, intercepted between two parallel planes of distance 2 and 4 from the centre is. a) 20π b) 40π c) 10π d) 30π

6 Integrating factor of $\frac{dy}{dx}+\frac{1}{x\log x}.y=\frac{2}{x^2}$ is a) e^x. b) log x. c) 1/ x d) e^{-x}.

7 The amount present in a radio active element disintegrates at a rate proportional to its amount . The differential equation corresponding to the above statement is (k is negative)

a) $\frac{dp}{dt}=\frac{k}{p}$ b) $\frac{dp}{dt}=kt$ c) $\frac{dp}{dt}=kp$ d) $\frac{dp}{dt}=-kt$

8 If $f'(x)=\sqrt{x}$ and f(1) = 2 then f(x) is.. a) $-\frac{2}{3}(x\sqrt{x}+2)$ b) $\frac{3}{2}(x\sqrt{x}+2)$ c) $\frac{2}{3}(x\sqrt{x}+2)$ d) $\frac{2}{3}x(\sqrt{x}+2)$

9 If A and B are any two matrices such that AB = 0 and A is non-singular, then

a) B = 0 b) B is singular c) B is non-singular d) B = A

10 If $ae^x+be^y=c$; $pe^x+qe^y=d$ and $\Delta_1=\begin{vmatrix} a & b \\ p & q \end{vmatrix}$; $\Delta_2=\begin{vmatrix} c & b \\ d & q \end{vmatrix}$; $\Delta_3=\begin{vmatrix} a & c \\ p & d \end{vmatrix}$ then the value of (x, y) is

a) $\left(\frac{\Delta_2}{\Delta_1},\frac{\Delta_3}{\Delta_1}\right)$ b) $\left(\log\frac{\Delta_2}{\Delta_1},\log\frac{\Delta_3}{\Delta_1}\right)$ c) $\left(\log\frac{\Delta_1}{\Delta_3},\log\frac{\Delta_1}{\Delta_2}\right)$ d) $\left(\log\frac{\Delta_1}{\Delta_2},\log\frac{\Delta_1}{\Delta_3}\right)$

11 The area of the parallelogram having a diagonal $3\vec{i}+\vec{j}-\vec{k}$ and a side $\vec{i}-3\vec{j}+4\vec{k}$ is,

a) $10\sqrt{3}$ b) $6\sqrt{30}$ c) $\frac{3}{2}\sqrt{30}$ d) $3\sqrt{30}$

12 The shortest distance of the point (2, 10, 1) from the plane $\vec{r}\cdot(3\vec{i}-\vec{j}+4\vec{k})=2\sqrt{26}$ is a) $2\sqrt{26}$ b) $\sqrt{26}$ c) 2 d) $\frac{1}{\sqrt{26}}$

13 The point of intersection of the line $\vec{r}=(\vec{i}-\vec{k})+t(3\vec{i}+2\vec{j}+7\vec{k})$ and the plane $\vec{r}\cdot(\vec{i}+\vec{j}-\vec{k})=8$ is

a) (8, 6, 22) b) (-8, - 6, -22) c) (4, 3, 11) d) (-4, -3, -11)

14 The shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is, a) $\frac{2}{\sqrt{3}}$ b) $\frac{1}{\sqrt{6}}$ c) $\frac{2}{3}$ d) $\frac{1}{2\sqrt{6}}$

15 If P represents the variable complex number z and if $|2z-1|=2|z|$ then the locus of P is

a) the straight line $x=\frac{1}{4}$ b) the straight line $y=\frac{1}{4}$ c) the straight line $z=\frac{1}{2}$ d) the circle $x^2+y^2-4x-1=0$

16 If $x=\cos\theta+i\sin\theta$ then the value of $x^n+\frac{1}{x^n}$ is a) $2\cos n\theta$ b) $2i\sin n\theta$ c) $2\sin n\theta$ d) $2i\cos n\theta$

17 If $\frac{1-i}{1+i}$ is a root of $ax^2+bx+1=0$, where a, b are real then (a,b) is a) (1,1) b) (1,-1) c) (0,1) d) (1,0)

18 The length of the latus rectum of the parabola whose vertex (2,-3) and the directrix x = 4 is.

a) 2 b) 4 c) 6 d) 8

19 The sum of the distance of any point on the ellipse $4x^2+9y^2=36$ from $(\sqrt{5},0)$ and $(-\sqrt{5},0)$ is.

a) 4 b) 8 c) 6 d) 18

20 If I is the unit matrix of order n, where $k \neq 0$ is a constant, then adj(kI) is

a) $k^n adj(I)$ b) $k\ adj(I)$ c) $k^2 adj(I)$ d) $k^{n-1} adj(I)$

21. If 2 cards are drawn from a well shuffled pack of 52 cards, the probability that they are of the same colours is.

a) 1 / 2. b) 26 / 51 c) 25 /51 d) 25 /102

22 If $\rho(A)=r$ then which of the following is correct ?

a) all the minors of order r which does not vanish b) A has atleast one minor of order r which does not vanish

c) A has atleast one (r+1) order minor which vanishes d) all (r+1) and higher order minors should not vanish

23 Chord AB is a diameter of the sphere $\left|\vec{r}-\left(2\vec{i}+\vec{j}-6\vec{k}\right)\right|=\sqrt{18}$ with coordinate of A as (3,2,-2) The coordinates of B is

a) (1,0,10) b) (-1,0,-10) c) (-1,0,10) d) (1,0,-10)

24 The work done by the force $\vec{F}=a\vec{i}+\vec{j}+\vec{k}$ in moving the point of application from (1,1,1) to (2,2,2) along a straightline is given to be 5 units. The value of a is a) -3 b) 3 c) 8 d) -8

25 Which of the following are correct ?

i) $\text{Re}(Z) \leq |Z|$ ii) $\text{Im}(Z) \geq |Z|$ iii) $|\overline{Z}| = |Z|$ iv) $\overline{(Z^n)} = (\overline{Z})^n$

a) (i) , (ii) b) (ii), (iii) c) (ii),(iii) and (iv) d) (i),(iii) and (iv)

26 For what value of x is the rate of increase $x^3 -2x^2 +3x+8$ is twice the rate of increase of x .

a) $\left(-\frac{1}{3},-3\right)$ b) $\left(\frac{1}{3},3\right)$ c) $\left(-\frac{1}{3},3\right)$ d) $\left(\frac{1}{3},1\right)$

27 Which of the following curves is concave down ? a) $y = -x^2$ b) $y = x^2$ c) $y = e^x$. d) $y = x^2 + 2x - 3$.

28. The curve $y = ax^3 + bx^2 + cx + d$ has a point of inflexion at x = 1 then

a) a+b = 0 b) a+ 3b = 0 c) 3 a + b = 0 d) 3a + b = 1

29. The directrix of the hyperbola $x^2-4(y-3)^2=16$ is

a) $y=\pm\frac{8}{\sqrt{5}}$ b) $x=\pm\frac{8}{\sqrt{5}}$ c) $y=\pm\frac{\sqrt{5}}{8}$ d) $x=\pm\frac{\sqrt{5}}{8}$

30 X is a discrete random variable which takes the values 0,1,2 and P(X =0) = 144 / 169, P(X=1) = 1/169 then the value of P(X=2) is. a) 145 / 169 b) 24 / 169 c) 2 / 169 d) 143 /169

31. $\mu_2 = 20$, $\mu_2' = 276$ for a discrete random variable X. Then the mean of the random variable X is.

a) 16 b) 5 c) 2 d) 1

32 The conditional statement $p \to q$ is equivalent to a) p v q. b) p v ~ q c) ~p v q. d) p ^ q.

33 The order of [7] in (Z_9, $+_9$) is... a) 9 b) 6 c) 3 d) 1.

34. Which of the following is correct ?

i. an element of a group can have more than one inverse.

ii. if every element of a group is its own inverse, then the group is abelian.

iii. the set of all 2 x 2 real matrices forms a group under matrix multiplication.

iv. $(a*b)^{-1} = a^{-1} * b^{-1}$ for all a, b $\in$ G

35 The condition that the line $lx+my+n=0$ may be a normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

a) $al^3+2alm^2+m^2n=0$ b) $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{(a^2+b^2)^2}{n^2}$ c) $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{(a^2-b^2)^2}{n^2}$ d) $\frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{(a^2+b^2)^2}{n^2}$

36 A continuous graph y = f (x) is such that $f'(x)\to\infty$ as $x\to x_1$, at (x_1, y_1) Then y = f (x) has a

a) vertical tangent $y = x_1$ b) horizontal tangent $x = x_1$

c) vertical tangent $x = x_1$ d) horizontal tangent $y = y_1$

37 $I_n = \int \cos^n x\, dx$ then $I_n =$

a) $-\frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}$ b) $\cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}$

c) $\frac{1}{n}\cos^{n-1} x \sin x - \frac{n-1}{n} I_{n-2}$ d) $\frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}$

38 The order and degree of the differential equation are $y = 4\frac{dy}{dx} + 3x\frac{dx}{dy}$

a) 2, 1 b) 1 ,2 c) 1 ,2 d) 2,2

39. If p is true and q is false then which of the following statements is not true ?

a) $p \to q$ is false b) $p \vee q$ is true c) $p \wedge q$ is false d) $p \leftrightarrow q$ is true

40 If X is a continuous random variable then which of the following is incorrect?

a) $F'(x) = f(x)$ b) $F(\infty) = 1$; $F(-\infty) = 0$

c) $P[a \le x \le b] = F(b) - F(a)$ d) $P[a \le x < b] \ne F(b) - F(a)$

PART – B

Note (i) Answer any ten questions

(ii) Question Number 55 is compulsory and choose any nine questions from the remaining

10×6=60

41 Solve by matrix inversion method each of the following system of linear equations: $2x - y = 7$, $3x - 2y = 11$

42 If $A = \begin{bmatrix} -1 & 2 \\ 1 & -4 \end{bmatrix}$, verify the result $A(adj\ A) = (adj\ A)A = |A|\, I_2$

43 (i) The volume of a parallelepiped whose edges are represented by $-12\vec{i} + \lambda\vec{k}$, $3\vec{j} - \vec{k}$, $2\vec{i} + \vec{j} - 15\vec{k}$ is 546. Find the value of λ .

(ii) Find the angle between the line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{-2}$ and the plane $3x + 4y + z + 5 = 0$

44 Find the vector and Cartesian equation of the sphere whose centre is (1,2,3) and which passes through the point (5,5,3)

45 Simplify: $\frac{(\cos\alpha + i\sin\alpha)^3}{(\sin\beta + i\cos\beta)^4}$

46 For any two complex numbers Z_1, Z_2, show that (i) $\left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|}$ (ii) $\arg\left(\frac{Z_1}{Z_2}\right) = \arg(Z_1) - \arg(Z_2)$

47 Determine the points of inflection if any, of the function $y = x^3 - 3x + 2$

48 If $V = ze^{ax+by}$ and z is a homogenous function of degree n in x and y prove that $x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} = (ax + by + n)V$.

49 Evaluate $\int_0^1 \log\left(\frac{1}{x} - 1\right) dx$

50 Solve $(1 + x^2)\frac{dy}{dx} + 2xy = \cos x$

51 Show that $p \leftrightarrow q \equiv ((\sim p) \vee q) \wedge ((\sim q) \vee p)$

52 State and prove cancellation laws on groups.

53 Find the mean and variance of the distribution $f(x) = \begin{cases} 3e^{-3x} & , 0 < x < \infty \\ 0 & , elsewhere \end{cases}$

54 In a Poisson distribution if $P(X = 2) = P(X = 3)$ find $P(X = 5)$ $[given\ \ e^{-3} = 0.050]$.

55 (a) The tangent at any point of the rectangular hyperbola $xy = c^2$ makes intercepts a, b and the normal at the point makes intercepts p,q on the axes. Prove that $ap + bq = 0$

(or)

(b) Obtain the Maclaurin's Series for $\log_e (1+x)$

PART – C

Note (i) Answer any ten questions

(ii) Question Number 70 is compulsory and choose any nine questions from the remaining

$10\times 10=100$

56 Solve by matrix inversion method eachof the following system of linear equations:
$x+y+z=9, \quad 2x+5y+7z=52, \quad 2x+y-z=0$.

57 If $\vec{a}=2\vec{i}+3\vec{j}-\vec{k}$, $\vec{b}=-2\vec{i}+5\vec{k}$, $\vec{c}=\vec{j}-3\vec{k}$ Verify that $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$

58 Find the vector and Cartesian equation of the plane containing the line $\frac{x-2}{2}=\frac{y-2}{3}=\frac{z-1}{3}$ and parallel to the line $\frac{x+1}{3}=\frac{y-1}{2}=\frac{z+1}{1}$.

59 If P represents the variable complex number z. Find the locus of P, if $\arg\left(\frac{z-1}{z+3}\right)=\frac{\pi}{2}$

60 Find the eccentricity, centre, foci, vertices of the following ellipses and draw the diagram:
$x^2+4y^2-8x-16y-68=0$

61 A comet is moving in a parabolic orbit around the sun which is at the focus of a parabola. When the comet is 80 million kms from the sun, the line segment from the sun to the comet makes an angle of $\frac{\pi}{3}$ radians with the axis of the orbit. Find (i) the equation of the comet's orbit (ii) how close does the comet nearer to the sun?(Take the orbit as open rightward).

62 Gravel is being dumped from a conveyor belt at a rate of 30 ft3 / min and its coarsened such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?

63 A farmer has 2400 feet of fencing and want to fence of a rectangular field that borders a straight river. He needs no fence along the river. What ar the dimensions of the field that has the largest area?

64 If $w=u^2 e^v$ where $u=\frac{x}{y}$ and $v=y\log x$, find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$

65 Find the area between the line $y=x+1$ and the curve $y=x^2-1$.

66 Show that the surface area of the solid obtained by revolving the arc of the curve $y=\sin x$ from $x=0$ to $x=\pi$ about x-axis is $2\pi\left[\sqrt{2}+\log(1+\sqrt{2})\right]$

67 Solve : $(2\sqrt{xy}-x)dy+y\,dx=0$

68 Show that the set $G=\{a+b\sqrt{2} \ / \ a, b\in Q\}$ is an infinite abelian group with respect to addition.

69 The mean weight of 500 male students in a certain college in 151 pounds and the standard deviation is 15 pounds. Assuming the weights are normally distributed, find how many students weigh (i) between 120 and 155 pounds (ii) more than 185 pounds.

70 (a) Prove that the line $5x+12y=9$ touches the hyperbola $x^2-9y^2=9$ and find its point of contact. (

or)

(b) A drug is excreted in a patients urine. The urine is monitored continuously using a catheter. A patient is administered 10 mg of drug at time t = 0 , which is excreted at a Rate of $-3t^{1/2}$ mg/h.(i) What is the general equation for the amount of drug in the patient at time t > 0 ?(ii) When will the patient be drug free?